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Sorokin, Vladislav

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On the response of a nonlinear parametric amplifier driven beyond resonance

Vladislav Sorokin*,**

*Department of Mechanical Engineering, Technical University of Denmark, Kgs. Lyngby, Denmark

**Institute of Problems in Mechanical Engineering Russian Academy of Science, St.Petersburg, Russia

Summary. The present paper is concerned with analysis of a nonlinear parametric amplifier response in a broad range of system parameters, particularly beyond resonance. It is obtained that amplitude of the amplifier steady-state response may reach large values in the case of arbitrarily small external excitation, so that the amplifier gain tends to infinity. A very large gain may be achieved in a broad range of parameters, particularly at small values of the parametric excitation amplitude.

Introduction

Classical parametric amplifiers are linear systems under the action of combined external and parametric excitation. The dynamics of such systems is thoroughly studied (see, e.g. [1]). However, many small-scale parametric amplifiers based on resonant micro- and nanosystems exhibit a distinctly nonlinear behavior when amplitude of their response is sufficiently large. So, it becomes necessary to consider such systems dynamics in a nonlinear context. The Duffing-type nonlinearity may serve as the simplest model. For example, in paper [2] the near-resonant response of such system was studied at small values of the parametric excitation amplitude and the nonlinearity coefficient. In the present paper this system is considered in a broader range of parameters. We abandon the requirement for the natural frequency of the corresponding linearized system to be close to the frequency of external excitation, and do not consider the parametric excitation amplitude and the nonlinearity coefficient as necessarily small. So, the following equation is studied:

$$z'' + \gamma z' + \delta z + \chi z \cos 2t_0 + kz^3 = A\cos(t_0 + \phi) \tag{1}$$

Here z represents the amplifier response, γ is coefficient of dissipation, which is assumed to be linear, A and χ are the amplitudes of external and parametric excitations correspondingly, ϕ is relative phase term, δ is the squared normalized natural frequency of the linearized system, and t_0 is dimensionless time.

Solution by the modified method of direct separation of motions

For studying equation (1) the modified method of direct separation of motions (MDSM) is employed. Conventional MDSM is a method which facilitates solution of various problems involving action of high-frequency vibrations on nonlinear mechanical systems (see, e.g. [3]). The modified MDSM is adapted for analysis of equations without a small parameter [4]. It implies solutions to be sought in the form:

$$z = \alpha(T_1) + \psi(T_1, T_0) \tag{2}$$

Where $T_0 = t_0$, $T_1 = \varepsilon T_0$, $\varepsilon << 1$ is an artificial small parameter, α is “slow”, and ψ is “fast”, 2π- periodic in dimensionless time T_0 variable, with average zero: $\langle \psi(T_1, T_0)\rangle = 0$. Here $\langle ... \rangle$ designates averaging in the period 2π on time T_0. The method may be interpreted in the following way [4]: It is applicable for solving equations without a small parameter; however, there is a fee one has to pay for this. The fee lies in the imposition of a certain restriction on the sought solutions: only solutions which are close to periodic may be determined by the means of the method. The restriction expresses itself in the introduction of the artificial small parameter ε, which defines the proximity of the sought solution to a periodic one. When this parameter tends to zero, we get pure periodic in time t_0 solution.

By averaging equation (1) on time T_0 obtain the following equation of “slow” motion (for variable α)

$$\varepsilon^2 \frac{d^2\alpha}{dT_1^2} + \varepsilon\gamma \frac{d\alpha}{dT_1} + \delta\alpha + \chi \langle \psi \cos 2T_0 \rangle + k(\alpha^3 + 3\alpha \langle \psi^2 \rangle + \langle \psi^3 \rangle) = 0 \tag{3}$$

Equation of “fast” motions (for variable ψ) may be obtained by subtracting equation (3) from equation (1)

$$\begin{aligned} &\frac{\partial^2\psi}{\partial T_0^2} + 2\varepsilon \frac{\partial^2\psi}{\partial T_1 \partial T_0} + \varepsilon^2 \frac{\partial^2\psi}{\partial T_1^2} + \gamma\left(\frac{\partial\psi}{\partial T_0} + \varepsilon\frac{\partial\psi}{\partial T_1}\right) + k\left(\psi^3 + 3\alpha\psi^2 + 3\alpha^2\psi - 3\alpha\langle\psi^2\rangle - \langle\psi^3\rangle\right) + \\ &+\delta\psi = -\chi\left((\alpha+\psi)\cos 2T_0 - \langle\psi\cos 2T_0\rangle\right) + A\cos(T_0 + \phi) \end{aligned} \tag{4}$$

Taking into account that ψ is a time T_0 periodic function, solution of equation (4) is sought in the form of series

$$\psi = B_1(T_1)\cos\left(T_0 + \theta_1(T_1)\right) + B_2(T_1)\cos\left(2T_0 + \theta_2(T_1)\right) + ... \tag{5}$$

For our purpose it is sufficient to retain only the first harmonic in (5). The accounting of other harmonics is not difficult, but leads only to a minor quantitative change of the results. Stable steady-state response of the amplifier is of primary interest. So, in order to compose equations that would describe it substitute (5) into (3) and (4), and require the coefficients of $\cos\left(T_0+\theta_1(T_1)\right)$ and $\sin\left(T_0+\theta_1(T_1)\right)$ in (4) to vanish. Consequently, letting all the derivatives with respect to times T_0 and T_1 to be equal to zero, obtain:

$$\delta\alpha + k(\alpha^3 + \tfrac{3}{2}\alpha B_1^2) = 0\,, \tag{6}$$

$$-B_1 + \delta B_1 + \tfrac{3}{4}kB_1^3 + 3k\alpha^2 B_1 = -\tfrac{1}{2}\chi B_1 \cos 2\theta_1 + A\cos(\theta_1 - \phi) \tag{7}$$

$$\gamma B_1 = \tfrac{1}{2}\chi B_1 \sin 2\theta_1 - A\sin(\theta_1 - \phi)\,, \tag{8}$$

In the present paper relations $\delta > 0$, $k > 0$ are considered as fulfilled, so equation (6) has single real solution $\alpha = 0$. From the derived equation of slow motion (3) it follows that this solution is stable. Examine the case of negligible small amplitude of external excitation: $A \sim \varepsilon^2$. As the result, for amplitude B_1 obtain the following expressions:

$$B_1 = 0\,, \qquad B_1 = \sqrt{\frac{4}{3k}\left(\pm\sqrt{\frac{1}{4}\chi^2 - \gamma^2} + 1 - \delta\right)} \tag{9}$$

So, when relations $\chi > 2\gamma$, $\sqrt{\tfrac{1}{4}\chi^2 - \gamma^2} + 1 - \delta > 0$ hold true, stable oscillations with amplitude $B_1 = \sqrt{\tfrac{4}{3k}\left(\sqrt{\tfrac{1}{4}\chi^2 - \gamma^2} + 1 - \delta\right)}$ may arise in the considered system even for an arbitrarily small value of the external excitation amplitude A. The stability of these oscillations follows from the fast motions equation (4). As an illustration, the dependencies of the steady-state amplitude B_1 of the nonlinear parametric amplifier response on parameter δ are shown in Figure 1. Solid lines correspond to stable branches, and dashed lines to unstable branches.

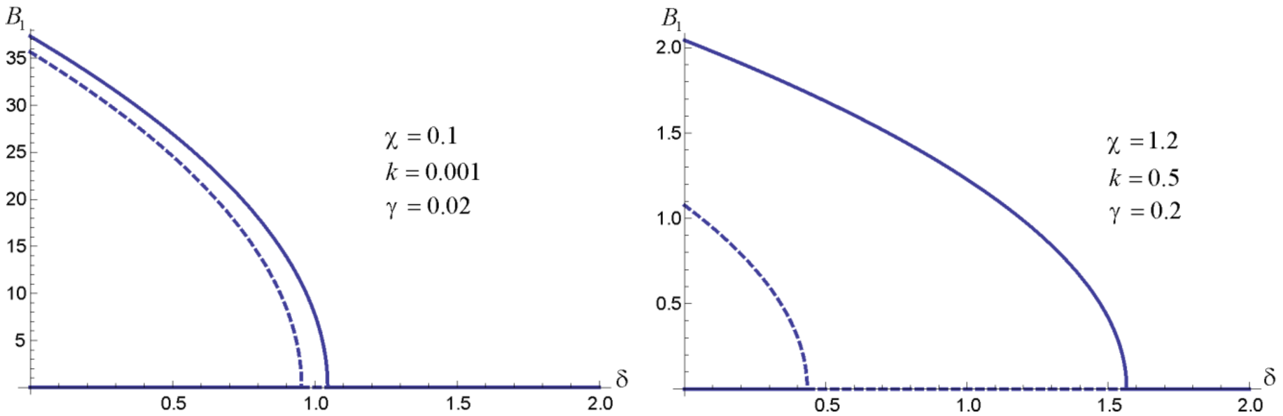

Figure 1. The dependencies of the steady-state amplitude of the nonlinear parametric amplifier response on parameter δ. Solid lines correspond to stable branches, and dashed lines to unstable branches.

So, the amplifier gain $G = B_1 \big/ B_1\big|_{\chi=0}$ tends to infinity when the amplitude of external excitation is negligibly small. As follows from expression (9) and Figure 1, the range of parameters δ and χ in which the gain is very large turns out to be significantly broader than the one of the linear amplifier.

Conclusions

In the present paper the response of a nonlinear parametric amplifier is studied in a broad range of system parameters, particularly beyond resonance. It is shown that the amplitude of this response may reach large values in the case of arbitrarily small external excitation, so that the amplifier gain tends to infinity. A very large amplifier gain may be achieved in a broad range of parameters, in particular when the amplitude of parametric excitation is comparatively small. The obtained results clearly demonstrate that very meaningful parametric amplification can be realized in resonant systems driven within a nonlinear response regime, and that nonlinear parametric amplifier possesses certain advantages over linear one. The work is financed by the Danish Council for Independent Research and FP7 Marie Curie Actions – COFUND: DFF – 1337-00026.

European Commission

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